Dynamic risk indifference pricing in incomplete markets

Current challenges in finance: New theoretical approaches in financial and banking risk management

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Abstract

- I investigate a dynamic pricing formula for contingent claims in incomplete markets based on the risk indifference principle.

Incompleteness

- The incompleteness comes from the illiquidity of the underlying *traded* assets.
- The number of risky assets is smaller than the dimension of the BM which models the risk factors on the market.
- Untraded source of uncertainty: international risk sharing, market capitalization range index returns, unspanned volatility, etc.
- More generally: any additional source of friction which creates illiquidity.

Mathematics

- Using a dual characterization of dynamic risk measures, the risk indifference pricing problem reduces to two (zero-sum) stochastic differential games, which I solve by means of BSDE theory.
- Non-Markovian time-consistent framework; dynamic risk measures from BMO martingales.
- Extends to the jump diffusion case.
- Øksendal and Sulem (2009); Bion-Nadal (2008); El Karoui, Peng and Quenez (1997); Hamadène and Lepeltier (1995).

Results

- Linear representation of the price as the expected derivative payoff under the risk indifference martingale measure.
- Dynamic risk indifference approach provides tighter price bounds than upper and lower hedging prices.

Applications

- Develop a methodology for determining a priori risk specific asset price bounds when markets are incomplete.
- Reference seller and buyer prices with respect to a predetermined institution-specific (or regulatory) measure of risk.
- Information on the risk sensitivity of financial products.
- Quantification of the risk associated with completeness assumptions.

The pricing principle

- In an arbitrage-free complete market, ∃! equivalent martingale measure (EMM) for pricing contingent claims.
- In incomplete markets, there is no unique EMM.
- Two major approaches exist in the litterature (*Xu, 2005*):
  - Pick a specific martingale measure for pricing according to some optimality criterion
  - Utility-based derivative pricing
- Here, maximising utility ⇔ minimizing risk exposure:
  - More often used in practice
  - Preserves the advantage of utility pricing (economic justification) and yields explicit solutions in general cases (beyond exponential utility models)
  - Natural extension to the Black-Scholes formula

Utility indifference pricing

Suppose the interest rate is zero, π is a portfolio, and
$X_x^{(\pi)}(t) = x + \int_0^t \pi(s) dS_s / S_s$ is a self-financing wealth process.

(i) If a person sells a contract which guarantees a payoff $G(\omega) \in L^\infty(\Omega, \mathcal{F}_T, P)$ at time T and receives a payment p_t for this, then at time t the maximal expected utility for the seller is

$$V_t^G(x + p_t) = \sup_{\pi \in \Pi} E[U_t(X_{x+p_t}^{(\pi)}(T) - G)].$$

(ii) If, on the other hand, no contract is sold, and hence no payment is received, then at time t the maximal expected utility for the person is

$$V_t^0(x) = \sup_{\pi \in \Pi} E[U_t(X_x^{(\pi)}(T))].$$

The (seller's) utility indifference price $p_t^{risk} = p_t$ of the claim G is the solution of the equation $V_t^G(x + p_t) = V_t^0(x)$ for every time $t \in [0, T]$.

Conditional convex risk measures

A convex risk measure $\rho_{i,j}$ on $(\Omega, \mathcal{F}_j, P)$ conditional to $(\Omega, \mathcal{F}_i, P)$, with $\mathcal{F}_i \subset \mathcal{F}_j$, is a map $\rho_{i,j} : L^\infty(\Omega, \mathcal{F}_j, P) \to L^\infty(\Omega, \mathcal{F}_i, P)$ s.t.:

- **Monotonicity**: $\forall X, Y \in L^\infty(\Omega, \mathcal{F}_j, P)$, if $X \leq Y$, then

$$\rho_{i,j}(X) \geq \rho_{i,j}(Y).$$

- **Translation invariance**: $\forall Z \in L^\infty(\Omega, \mathcal{F}_i, P), \forall X \in L^\infty(\Omega, \mathcal{F}_j, P)$,

$$\rho_{i,j}(X + Z) = \rho_{i,j}(X) - Z.$$

- **Convexity**: $\forall X, Y \in L^\infty(\Omega, \mathcal{F}_j, P), \forall\ 0 \leq \lambda \leq 1$,

$$\rho_{i,j}(\lambda X + (1-\lambda)Y) \leq \lambda \rho_{i,j}(X) + (1-\lambda)\rho_{i,j}(Y).$$

Additional properties

A conditional convex risk measure can have additional properties:

- **Continuity from below**: For any increasing sequence X_n of elements of $L^\infty(\Omega, \mathcal{F}_j, P)$ such that $X = \lim X_n$ *a.s.*, the sequence $\rho_{i,j}(X_n)$ has the limit $\rho_{i,j}(X)$ *a.s.* $\Rightarrow$ dual representation
- **Normalization**: $\rho_{i,j}(0) = 0 \Rightarrow \rho_{i,j}(X) \in L^\infty(\Omega, \mathcal{F}_i, P)$.

Many references: *Artzner et al.; Barrieu and El Karoui; Bion-Nadal; Cheridito, Delbaen and Kupper; Detlefsen and Scandolo; Frittelli and Gianin; Klöppel and Schweizer; Peng; etc.*

Dynamic risk indifference pricing

(i) If a person sells a contract which guarantees a payoff $G(\omega) \in L^\infty(\Omega, \mathcal{F}_T, P)$ at time T and receives a payment p_t for this, then at time t the minimal risk involved for the seller is

$$\Phi_t^G(x + p_t) = \inf_{\pi \in \Pi} \rho_t(X_{x+p_t}^{(\pi)}(T) - G).$$

(ii) If, on the other hand, no contract is sold, and hence no payment is received, then at time t the minimal risk for the person is

$$\Phi_t^0(x) = \inf_{\pi \in \Pi} \rho_t(X_x^{(\pi)}(T)).$$

The (seller's) risk indifference price $p_t^{risk} = p_t$ of the claim G is the solution of the equation $\Phi_t^G(x + p_t) = \Phi_t^0(x)$ for every time $t \in [0, T]$.

Dual representation of convex risk measures

It is well-known that a dynamic convex risk measure can be represented as follows:

$$\rho_t(X) = \operatorname*{ess\,sup}_{Q \in M} \left\{ E_Q[-X|\mathcal{F}_t] - \zeta_t(Q) \right\},$$

where M is a family of measures and ζ is a “penalty function” satisfying appropriate assumptions.

Zero-sum stochastic differential games

Using the dual representation, the problem of finding the risk indifference price $p_t = p_{risk}$ amounts to solving 2 zero-sum stochastic differential game problems:

Find $\Phi_t^G(x + p_t)$ and an optimal pair $(\pi^*, Q^*) \in \Pi \times M$ such that

$$\Phi_t^G(x + p_t) = \operatorname*{ess\,inf}_{\pi \in \Pi} \sup_{Q \in M} \left\{ E_Q[-X_{x+p_t}^{(\pi)}(T) + G | \mathcal{F}_t] - \zeta_t(Q) \right\}$$

and

$$\Phi_t^0(x) = \operatorname*{ess\,inf}_{\pi \in \Pi} \sup_{Q \in M} \left\{ E_Q[-X_x^{(\pi)}(T) | \mathcal{F}_t] - \zeta_t(Q) \right\},$$

for $t \in [0, T]$ and for a given family of measures M and a given penalty function ζ.

The model

The financial market consists of

- one riskless asset with price constant at 1 (zero interest rate)
- k risky assets evolving according to the SDE

$$dS_t^i = S_t^i(\mu_t^i dt + \sigma_t^i dW_t), \qquad t \leq T, \qquad S_0^i > 0, \qquad 1 \leq i \leq k,$$

where μ_t a $\mathcal{F}_t$-predictable vector-valued map $\mu : [0, T] \to \mathbb{R}^k$ and σ_t a $\mathcal{F}_t$-predictable matrix-valued map $\sigma : [0, T] \to \mathbb{R}^{k \times m}$ s.t.
$\int_0^T |\mu_s| + |\sigma_s|^2 \, ds < \infty, a.s.$ The BM is a $(m, 1)$ vector.
I assume the AOA in the financial market and $m > k$.

Self-financing trading strategy

A portfolio in this market is represented by the $(1, k)$ row vector $\pi(t)$ standing for the amount invested in the risky assets at time t. The dynamics of the corresponding (discounted) wealth $X(t) = X^{(\pi)}(t)$ is

$$\begin{aligned} dX(t) &= \pi(t)dS(t)/S(t) = \pi(t)\left[\mu(t)dt + \sigma(t)dW(t)\right]; \qquad t \in [0, T] \\ X(0) &= x > 0. \end{aligned}$$

The portfolio $\pi(t)$ is admissible if it is predictable and satisfies

$$\int_0^T \left(|\pi(t)\mu(t)|^2 + |\pi(t)\sigma(t)|^2\right) dt < \infty, a.s.$$

and $X^{(\pi)}(t) \geq 0$ for any $t \leq T$, *a.s.* I assume that $\exists\ C \in \mathbb{R}$ s.t. $X^{(\pi)}(T) \leq C$ a.s.
The set of all admissible portfolios is denoted by Π.

A set of measures

A natural choice of the family M is the set of measures $Q = Q_\theta$ of Girsanov transformation type. For a given process $(\theta(t))_{t \leq T}$, define $K_\theta(t)$ as the solution of

$$\begin{aligned} dK_\theta(t) &= K_\theta(t)\left[\theta(t)dW(t)\right], \qquad t \in [0, T] \\ K_\theta(0) &= k > 0, \end{aligned}$$

where θ_t is assumed to be in BMO, i.e., it is a $\mathcal{F}_t$-predictable $\mathbb{R}^m$-valued process such that there is a constant C which, for any stopping time $\tau \leq T$, satisfies

$$E\left[\int_\tau^T |\theta_s|^2 \, ds | \mathcal{F}_\tau\right] \leq C^2.$$

Then define the measure $Q = Q_\theta$ by

$$dQ_\theta(\omega) = K_\theta(T)dP(\omega) \text{ on } \mathcal{F}_T.$$

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A set of controls

Finally, let Θ be the set of all controls $\theta(t)$ such that $E\left[K_\theta(T)\right] = K_\theta(0) = 1$ and $L\theta(t) = 0$, where

$$L\theta(t) = \mu(t) + \sigma(t)\theta(t); \quad t \in [0, T].$$

Now define the set M of measures as follows:

$$M_T = \{Q_\theta; \theta \in \Theta\}.$$

The second condition and Girsanov theorem imply that all the measures $Q_\theta \in M$ are EMM.

Controlled state process

Define the controlled process $Y(t) = Y^{\theta,\pi}(t) \in \mathbb{R}^{1+k=d}$ as follows:

$$dY(t) = \begin{bmatrix} dY_1(t) \\ dY_2(t) \end{bmatrix} = \begin{bmatrix} dK_\theta(t)/K_\theta(t) \\ dS(t)/S(t) \end{bmatrix} = \begin{bmatrix} 0 \\ \mu(t) \end{bmatrix} dt + \begin{bmatrix} \theta(t) \\ \sigma(t) \end{bmatrix} dW(t)$$

and

$$Y(0) = y = (k, s) \in \mathbb{R}^d.$$

Penalty function

We assume that the penalty function ζ has the form

$$\zeta_t(Q_\theta) = E_{Q_\theta}\left[\int_t^T \lambda(s, Y, \theta(s))ds + h(Y)|\mathcal{F}_t\right]$$

where λ and h are bounded functions with values in $\mathbb{R}^+$. These assumptions imply (Bion-Nadal, 2008) that

$$\rho_t(X) = \operatorname*{ess\,sup}_{Q\in M_T}\{E_Q[-X|\mathcal{F}_t] - \zeta_t(Q)\}$$

is a time-consistent dynamic convex risk measure.

Precise formulation of the problem

Assuming that the given claim G has the non-Markovian form $G = g(S)$ for some bounded real function g, we must find $\Phi_t^G(y)$ and an optimal pair $(\pi^*, \theta^*) \in \Pi \times \Theta$ such that

$$\Phi_t^G(y, x) = \Phi_t^G(k, s, x) := \operatorname*{ess\,inf}_{\pi \in \Pi} \left(\sup_{\theta \in \Theta} J_t^{\pi,\theta}(y) \right) = J_t^{\pi^*,\theta^*}(y), \qquad (1)$$

where

$$J_t^{\pi,\theta}(y) := E_{Q_\vartheta}\left[-\int_t^T \lambda(s, Y, \theta(s))ds - h(Y) - X_x^{(\pi)}(T) + g(S) | \mathcal{F}_t \right].$$

Linear BSDEs and conditional expectations

Theorem (El Karoui, Peng and Quenez, 1997)

Let (β, μ) be a bounded $(\mathbb{R}, \mathbb{R}^d)$-valued progressively measurable process, φ be an element of $H_1^2(0, T)$ and $\xi_T \in L_1^2(0, T)$. Then

$$-dY_t = (\varphi_t + \beta_t Y_t + \mu_t Z_t)dt - Z_t dW_t, \; Y_T = \xi$$

has a unique solution $(Y, Z) \in S_1^2(0, T) \times H_d^2(0, T)$ and Y is explicitly given by

$$Y_t = E\left[\xi\Gamma_{t,T} + \int_t^T \Gamma_{t,s}\varphi_s ds \Big| \mathcal{F}_t\right],$$

where $d\Gamma_{t,s} = \Gamma_{t,s}(\beta_s ds + \mu_s dW_s)$, $\quad \Gamma_{t,t} = 1$ satisfies the flow property $\Gamma_{t,s}\Gamma_{s,u} = \Gamma_{t,u}$, $\forall t \leq s \leq u$, P- a.s.

BSDEs and control problems

Suppose that R_t is the solution of a BSDE with coefficient f and terminal condition ξ depending on α. Then we have

$$\textit{ess}\inf\sup R_t(f^\alpha, \xi^\alpha) = R_t(\inf\sup f^\alpha, \inf\sup \xi^\alpha).$$

Optimality conditions

Define the functional H as

$$H(t, y, z, \theta_t) := -\lambda(t, y, \theta_t) + \overline{z}\theta_t,$$

where $\overline{z}$ is the transpose of $z \in \mathbb{R}^m$ and define

$$\Lambda(T, y, \theta_T, \pi_T) := g(S) - X_x^{(\pi)}(T) - h(Y).$$

We assume that there exists optimal strategies $\theta^* = \theta^*(t, y, z) \in \Theta$ and $\pi^* = \pi^*(t, y) \in \Pi$ such that

$$\begin{aligned} H(t, y, z, \theta_t) &\leq H(t, y, z, \theta_t^*), \\ \Lambda(T, y, \theta_T, \pi_T^*) &\leq \Lambda(T, y, \theta_T^*, \pi_T^*) \leq \Lambda(T, y, \theta_T^*, \pi_T). \end{aligned}$$

Solution of the game

Theorem

Assume the optimality conditions hold. Then there exists (R_t^*, Z_t^*) *solution of the BSDE:*

$$-dR_t^* = H(t, y, z, \theta^*)dt - Z_t^* dW_t, \qquad R_T^* = \Lambda(T, y, \theta_T^*, \pi_T^*).$$

In addition the pair of strategies (π^*, θ^*) *is a saddle-point for the zero-sum stochastic differential game and*

$$J_t(\pi^*, \theta^*) = R_t^* = \operatorname*{ess\,inf}_{\pi\in\Pi} \sup_{\theta\in\Theta} J_t(\pi, \theta),$$

and the intial value of the game is R_0^*.

Proof

Under the above assumption, the results of Briand and Confortola (2007) apply and

$$-dR^*_t = (-\lambda(t, y, \theta^*_t) + Z^*_t \theta^*_t) dt - Z^*_t dW_t, \qquad R^*_T = \Lambda(T, y, \theta^*_T, \pi^*_T)$$

has a unique solution $(R^*_t, Z^*_t) \in S^2_1(0, T) \times H^2_d(0, T)$. Moreover, R^*_t is explicitly given by

$$R^*_t = E\left[\Lambda(T, y, \theta^*_T, \pi^*_T)\Gamma^*_{t,T} - \int_t^T \Gamma^*_{t,s} \lambda(s, y, \theta^*_s) ds | \mathcal{F}_t\right],$$

where $(\Gamma^*_{t,s})_{s \geq t}$ is the adjoint process defined by the forward linear SDE $d\Gamma^*_t = \Gamma^*_t(\theta^*_t dW_t)$, $\qquad \Gamma^*_{t,t} = 1.$

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Proof (continued)

Using the flow property of $\Gamma^*_{t,s}$ and writing Γ^*_t for $\Gamma^*_{0,t}$ we have that

$$\Gamma^*_t R^*_t = E\left[\Lambda(T,y,\theta^*_T,\pi^*_T)\Gamma^*_T - \int_t^T \Gamma^*_s \lambda(s,y_\cdot,\theta^*_s)ds|\mathcal{F}_t\right].$$

Remembering that Γ^*_T defines the probability Q_{θ^*}, i.e., $dQ_{\theta^*} = \Gamma^*_T dP$, we have

$$\begin{aligned} J_t(\pi^*,\theta^*) &= E_{Q_{\vartheta^*}}\left[\Lambda(T,y,\theta^*_T,\pi^*_T) - \int_t^T \lambda(s,Y_\cdot,\theta^*_s)ds|\mathcal{F}_t\right] \\ &= \frac{E\left[\Gamma^*_T\Lambda(T,y,\theta^*_T,\pi^*_T) - \int_t^T \Gamma^*_s\lambda(s,Y_\cdot,\theta^*_s)ds|\mathcal{F}_t\right]}{\Gamma^*_t}. \end{aligned}$$

Hence we obtain that $J_t(\pi^*,\theta^*) = R^*_t$ *a.s.*

Proof (continued)

Using the comparison theorem for BSDEs and the optimality conditons, one can verify that the pair (π^*, θ^*) is a saddle-point for the game and that R_0^* is the initial value of the game. $\square$

A related stochastic control problem

Consider the problem of finding $\Psi_G(y)$ and an optimal control $\widehat{\theta} \in \Theta$ such that

$$\Psi_t^G(y) = \sup_{\theta \in \Theta} J_t^{\theta}(y) = J_t^{\widehat{\theta}}(y), \qquad (2)$$

where

$$\begin{aligned} J_t^{\theta}(y) &= J_t(\theta) := E_{Q_{\vartheta}}\left[-\int_t^T \lambda(s, Y, \theta_s) ds - h(Y) + g(S) | \mathcal{F}_t \right], \\ y &= (k, s) \in \mathbb{R}^{d-1}. \end{aligned}$$

Linearity in the wealth process

Theorem

Suppose $\Phi_t^G(x,y)$ *is the value function for the initial problem (1) and* $\Psi_t^G(y)$ *the value function for the last one, (2). Then we have that*

$$\Phi_t^G(x,y) = \Psi_t^G(y) - X_x^{(\pi^*)}(t),$$

for some optimal pair (θ^*, π^*)*.When* $t = 0$*, there exists an optimal* $\theta^* \in \Theta$ *for problem (2) such that for all* $\pi \in \Pi$ *the pair*

$$(\theta^*, \pi^*) = (\widehat{\theta}, \pi)$$

is an optimal pair for problem (1).

Proof

First we seek to characterize the solution of the stochastic control problem (2). Proceeding as in the preceding Theorem, we get $\Psi_G(y) = \widehat{R}_0$, with

$$-d\widehat{R}_t = (-\lambda(t, y, \widehat{\theta}_t) + \widehat{Z}_t\widehat{\theta}_t)dt - \widehat{Z}_t dW_t, \qquad \widehat{R}_T = g(S) - h(Y). \quad (3)$$

We can choose $\widehat{\theta}_t = \theta_t^*$ in order to satisfy the optimality conditions, because λ and h are independent of π. Note that for all $\pi \in \Pi$

$$E_{Q_{\theta^*}}\left[X_x^{(\pi)}(T)|\mathcal{F}_t\right] = X_x^{(\pi)}(t)$$

since Q_{θ^*} is an equivalent martingale measure. In particular, when $t = 0$, $E_{Q_{\theta^*}}\left[X_x^{(\pi)}(T)\right] = x$.
Comparing the BSDEs for problems (1) and (2), we see that $R_T^* = \widehat{R}_T - X_x^{(\pi^*)}(T)$.

Proof (continued)

$$\begin{aligned} J_t(\pi^*, \theta^*) &= E_{Q_{\vartheta^*}}\left[-\int_t^T \lambda(s, y, \theta_s^*) ds - h(\widetilde{Y}) - X_x^{(\pi^*)}(T) + g(S) | \mathcal{F}_t \right] \\ &= E_{Q_{\widehat{\theta}}}\left[-\int_t^T \lambda(s, y, \widehat{\theta}_s) ds - h(\widetilde{Y}) + g(S) | \mathcal{F}_t \right] - X_x^{(\pi^*)}(t) \\ &= J_t(\widehat{\theta}) - X_x^{(\pi^*)}(t). \end{aligned}$$

Hence $\Phi_t^G(y) = \Psi_t^G(y) - X_x^{(\pi^*)}$.When $t = 0$, this reduces to $\Phi_0^G(y) = \Psi_0^G(y) - x$ hence the claim follows. □

Dynamic risk indifference pricing

We now solve the initial risk indifference pricing equation, i.e., find $p_t = p_t^{risk}$ in

$$\Phi_t^G(k, s, x + p) = \Phi_t^0(k, s, x),$$

where Φ_t^G is the solution of problem (1). By the preceding Theorem, this reduces to

$$\Psi_t^G(k, s) - X_{x+p_t}^{(\pi^*)}(t) = \Psi_t^0(k, s) - X_x^{(\pi^*)}(t).$$

Hence,

$$p_t = \Psi_t^G(k, s) - \Psi_t^0(k, s). \tag{4}$$

A BSDE formulation

We can rewrite p_t as $p_t = R_t^G - R_t^0$, where

$$\begin{aligned} -dR_t^G &= (-\lambda(t,y,\theta_t^*) + Z_t^G \theta_t^*)dt - Z_t^G dW_t, \\ R_T^G &= g(S) - h(Y) \\ -dR_t^0 &= (-\lambda(t,y,\theta_t^*) + Z_t^0 \theta_t^*)dt - Z_t^0 dW_t, \\ R_T^0 &= -h(Y). \end{aligned}$$

Therefore,

$$dp_t = -dR_t^{risk} = Z_t^{risk}\theta_t^* dt - Z_t^{risk} dW_t, \qquad R_T^{risk} = g(S)$$

where $R_t^{risk} = R_t^G - R_t^0$ and $Z_t^{risk} = Z_t^G - Z_t^0$, and

$$p_t = E_{Q_{\theta^*}}[g(S)|\mathcal{F}_t].$$

Comparison with super hedging prices

Because the set of risk indifference EMM belongs to the set of EMM, we have

$$p_t^{risk/seller} \leq \sup_{Q_\theta \in EMM} E_{Q_\theta}[g(S)|\mathcal{F}_t] = p_t^{up},$$

where p_t^{up} is the superhedging price. More generally, one can proof the following

Corollary

$$p_t^{low}(G) \leq p_t^{risk/buyer}(G) \leq p_t^{risk/seller}(G) \leq p_t^{up}(G).$$

Future work

- Jump diffusion case.
- Determine an explicit risk indifference hedging strategy.
- Based on numerical simulations of BSDEs, assess the risk of various financial products, based on different risk measures.
- Conversely, compare, assess and design risk measures with respect to the size of the price interval that they induce.

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