Incomplete Markets and Asset Prices

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Introduction

We study option prices and the *volume* of traded options in a general equilibrium model.
Why is this important (new) ?

1. The volume of options traded contains information

Stylized fact : Positive correlation between derivative volumes and underlying variability.

- Shalen (JoF 1993), Chen, Cuny and Haugen (JoF, 1995) : correlation for future open interest.
- Burashi and Jilstov (JoF,2006) : correlation for option future open interest.
- $\Longrightarrow$ The more volatile the underlying asset, the larger the traded volume of the derivative.

2. Theoretical Interest : Measure of heterogeneity and Welfare

- Usual theory : Derivative pricing by replication (Black and Scholes, 1973) : No role for volumes.
  $\Rightarrow$ Market for the aggregate risk must be incomplete at least before the option introduction.
- Agents must value risk differently : Otherwise, they hold the same portfolio and no option trade occurs (No trade of insurance between agents)

Where does heterogeneity come from to understand option trading?

Heterogeneity in financial markets

- **Preferences :** Grossman and Zhou (JoF, 1996). Bates (JEDC, 2008), Weinbaum (JEDC, 2009), Bahmra and Uppal (RFS, 2007)
- **Information :** Detemple and Selden (IER, 1991) Burashi and Jiltsov (Jof, 2006)
- **Uninsurable (background) risk :** Franke, Stapleton and Subrahmanyam (JET, 1998).

We use uninsurable risks (incomplete insurance markets) to generate heterogeneity :

- Empirical support (Zeldes, 1989) and widely used in finance ; risk free rate (Hugett, 1993), equity premium (Constantinides and Duffie, 1996).
- Heterogeneity ex post : do not need different information or preferences.
- General consistency : incomplete market = only departure from standard complete market economies.

Contribution of the Paper

(First) Intertemporal model of derivative pricing with incomplete markets. General equilibrium model : both the underlying asset and options are prices. (Extend Challe, Le Grand and Ragot, 2008).

Results:

- Reproduce stylized fact about option volumes and volatility
- Welfare analysis of option introduction: who gains, who looses?

Plan of the Talk

1. Economy without options
2. Economy with options
3. A two agents economy
4. Conclusion

1 - The Economy without option

The Economy: Uncertainty and Heterogeneity

2 types of shocks: Aggregate and idiosyncratic.

- *Aggregate shock:* $z_t = \{z_1, \ldots, z_n\}$ is a first order MC with transition matrix $\Pi = (\pi_{i,j})_{1 \leq i,j \leq n}$. $(\sum_j \pi_{i,j} = 1)$
- Heterogeneity : I types of agents (mass 1 for each type):
- *Idiosyncratic shock:* $e_t^r = \{0, 1\}$ for agents r is a first order MC reflecting the individual employment status. Transition matrix T.

$$T = \begin{bmatrix} \alpha^i & 1 - \alpha^i \\ 1 - \rho^i & \rho^i \end{bmatrix}$$

Each agent suffers from an independent shock e_t^r.
When employed, agent of type i has the productivity $w_t^i = w^i(z_t)$.

Production:

- 1 single good produced with labor (no capital).
- A large number of firms employs agents with a real wage equal to their marginal productivity $w^i(z_t)$.

Asset Market: A single tree with a constant mass V.

- Payoffs in each period $y_t = y(z_t)$
- P_t = Price of one tree unit at t

An agent r of type $i = 1..I$

- When unemployed, earns δ? When employed, earns $w_t^i \, l_t^r$.
- Purchases (sells) the quantity x_t^r (x_{t-1}^r) of the tree for the price P_t. Borrowing constraints: $x_t^r \geq 0$.

Agent's r program:

$$\max_{c^r, l^r, x^r} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(u \left(c_{t+j}^r \right) - l_{t+j}^r \right)$$

$$\text{s.t.} \quad c_t^r + P_t \, x_t^r = e_t^r \, w_t^i \, l_t^r + (1 - e_t^r) \, \delta + (P_t + y_t) \, x_{t-1}^r$$

$$x_t^r \geq 0$$

Financial market equilibrium

$$\int_0^I x_t^r \, dr = V$$

In general, with incomplete markets: Continuous distributions (Bewley (1983), Aiygari(1994), Krussel and Smith (1998)).
Two important assumptions to decrease heterogeneity (Challe Ragot 2009; Challe, Le Grand and Ragot, 2008):

Endogenous Labor Supply Agents work (when employed) such that $u'(c_t^r) = 1/w_t^i$.
Portfolio Liquidation Unemployed households sell their whole portfolio and face credit constraints (would like to borrow)
Only employed agents hold securities.

For each type $i = 1..I$: a 4 agents' distribution (current and past situations): ee, ue, eu, uu, in constant proportions $\omega^{k,l}$ $(k, l = e, u)$.

Price of one unit of asset in state $k = 1..n$ is P_k
Euler equation ($k = 1, \ldots, n$ and $i = 1, \ldots, I$):

$$P_k \times \underbrace{\frac{1}{w_k^i}}_{\text{marginal utility}} = \underbrace{\alpha^i \beta \mathbb{E}_k \left[\underbrace{(P_{t+1} + y_{t+1})}_{\text{payoff}} \times \underbrace{\frac{1}{w_{t+1}^i}}_{\text{future marginal u.}} \right]}_{\text{smoothing term}}$$

$$\underbrace{+ (1-\alpha^i) \beta \mathbb{E}_k \left[\underbrace{(P_{t+1} + y_{t+1})}_{\text{payoff}} u' \underbrace{\left((P_{t+1} + y_{t+1}) \, x_k^i + \delta \right)}_{\text{value of liquidated portfolio}} \right]}_{\text{portfolio liquidation term}}$$

Pricing equation ($k = 1, \ldots, n$ and $i = 1, \ldots, I$):

$$P_k = \mathbb{E}_k m_k^i . \left(P_{t+1} + y_{t+1}\right)$$

with

$$m_k^i = \alpha^i \beta \frac{w_k^i}{w_{t+1}^i} + \left(1 - \alpha^i\right) \beta w_k^i u' \left(\left(P_{t+1} + y_{t+1}\right) x_k^i + \delta\right)$$

Equilibrium Existence: The main technical difficulty of the paper.
Agents have to liquidate their portfolio when unemployed.
Proven by the implicit function theorem.

2 - The Economy with option

In the previous economy, we introduce M call options (nothing but financial markets change):

- Differ only according to their strikes: $K_1 < \ldots < K_M$.
- Mature at the next date and payoff $(P_t - K_l)^+$
- At date t, an agent r purchases $s^r_{t,l}$ calls of strike K_l for the price $Q_{t,l}$.

Agents face 'Borrowing' constraints:
$\Longrightarrow$ Positive financial wealth: $P_t\, x^r_t + \sum_{l=1}^{M} s^r_{t,l} Q_{t,l} \geq 0$

Agent r (of type i)'s program:

$$\max_{c^r, l^r, x^r, (s_l^r)_{1 \leq l \leq M}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(u\left(c_{t+j}^r\right) - l_{t+j}^r \right)$$

Budget Constraint

$$c_t^r + P_t\, x_t^r + \sum_{l=1}^{M} s_{t,l}^r Q_{t,l} = e_t^r\, w_t^i\, l_t^r + (1 - e_t^r)\,\delta$$

$$+ (P_t + y_t)\, x_{t-1}^r + \sum_{l=1}^{M} (P_t - K_l)^+ \, s_{t-1,l}^r$$

Credit Constraint

$$P_t\, x_t^r + \sum_{l=1}^{M} s_{t,l}^r Q_{t,l} \geq 0$$

$(M+1)$ financial markets clear at all dates t:

$$\int_0^I x_t^r \, dr = V \quad \text{and} \quad \int_0^I s_{t,l}^r \, dr = 0 \quad (l = 1, \ldots, M)$$

Pricing Equations in state $k = 1..n$, $i = 1...I$

$$
\begin{aligned}
P_k &= \mathbb{E}_k \left[M^i_{k,j} \left(P_j + y_j \right) \right] \\
Q_{k,l} &= \mathbb{E}_k \left[M^i_{k,j} \left(P_j - K_l \right)^+ \right] \\
M^i_{k,j} &= \alpha^i \beta \frac{w^i_k}{w^i_j} \\
&\quad + \left(1 - \alpha^i \right) \beta w^i_k u' \left(\left(P_j + y_j \right) x^i_k + \sum_{m=1}^{M} \left(P_j - K_m \right)^+ s^i_{k,m} + \delta \right)
\end{aligned}
$$

$\Rightarrow N\, I$equalities

Equilibrium Existence: Proven by the implicit function theorem, strating from an economy without options.

Results in the General case

Two results regarding open interest: It converges to zero, when:
Uncertainty disappears: $y_k \longrightarrow y$
Heterogeneity vanishes: $w_k^i \longrightarrow w_k$
when $\alpha = 1$ (no idiosyncratic shock): volumes are not determined.
Option volumes reflect the interaction btw both.

3 - A Two Agents Economy

We restrict to the simplest case, where options can be traded $\Rightarrow$ derive analytical results on prices and volumes:

- Two agents with heterogeneous risk exposure $\alpha^1 > \alpha^2$: heterogeneity
- Two states of the world $y_G > y_B$: uncertainty (and market incompleteness)

Result 1 : Portfolio Allocation

The more "risky" (lower α) agent holds a larger quantity of assets
The more risky agent sell the call option to decrease the fluctuations in the value of the portfolio in case of liquidation.The other agents sell insurance in equilibrium.
Options are used as an insurance device and allows the wealthier agents to smooth his portfolio holdings.

Result 2 : Welfare analysis

After the option introduction: No effect at the first order: redistribution only.
Intuitions:
The risky agents trade options to insure against variation of value of their portfolio.
Because options exist: underlying asset more attractive : Increase further the volume asset held by the risky agent.
More risky agent gain from the option introduction whereas le less risky agent suffer from it.

Result 3 : Effect of volatility

When dividends are more volatile

The more risky agent purchases a larger asset quantity. . .

. . . and trade more options.

Asset prices become more volatile, and. . .

. . . option prices increase.

We are able to reproduce the link between aggregate volatility and options volume.

Conclusion

We propose a general equilibrium model of option pricing:
Options are traded at equilibrium
Plausible features
Analytical framework to study the impact of options trading.