

***Some thoughts  
about  
mathematizing  
risks***

- I. *The persistent question of the tails of probability distributions*
- II. *VaR vs Coherent measures of risk*
- III. *The financial crisis : a short analysis*
- IV. *Chance and meaning : two non-miscible fluids*
- V. *Back on tails of distributions*

# ***I. The persistent question of the tails of probability distributions***

- The discovery of the fact that the law  $dx/\pi(x^2+1)$  (today called Cauchy law) has no expectation and has the stable property, hence doesn't fulfilled the central limit theorem, goes back to Siméon Denis Poisson around 1830, twenty years before Cauchy.
- Henri Poincaré at the end of the nineteenth century notes that this type of distribution contradicts the least squares method.
- during the twentieth century, with the development of mathematical economics, operational research and decision theory, the quadratic mean reasonings expand in a whole general rationality
- if, for a decision in uncertainty, you choose the value of a parameter so that the mean of the squares of the discrepancies between this quantity and the chosen value be minimal, then this kind of choice is compatible with other subsequent choices taken with the same method and with possible sub-choices done along the same principle.

# I. *The persistent question of the tails of probability distributions*

- the use of utility functions, although yielding true services to represent the behaviour of the agents with their subjective probabilities (and avoid St Petersburg paradox), doesn't solve at all the problem of the tails of probability distributions.

- On one hand the reasonings in quadratic mean are iteratively coherent and develop in a whole rationality, on the other hand, procedures and arguments using thick probability tails build a language which tends to deteriorate

given a family of probability distributions  $P_\alpha$  on  $\mathbb{R}$ , each  $P_\alpha$  possessing an expectation, if the real parameter  $\alpha$  is random with law  $\nu(d\alpha)$ , it is possible that the resulting probability distribution

$$P = \int P_\alpha[\cdot] \nu(d\alpha)$$

have no expectation.

Climate models **Weitzman 2007**

# ***I. The persistent question of the tails of probability distributions***

**Other phenomena prevent from knowing the tails**

- histogramms**
- the «curse of dimensionality»**

**or make this knowledge sensitive to hypotheses**

- theorem of extreme values**

***II. Value at Risk***

***versus***

***Coherent measures of risk***

## II. *VaR vs coherent measures of risk*

- VaR (JP Morgan 1970's)  
maximal loss that is not exceeded with probability  $p$ .

VaR doesn't involve the tail of the distribution  
(practice  $\neq$  theory)

- the methodology VaR can be extended to any  
modelization : taking in account a boundary and the  
probability of remaining inside this domain

**BaR    Boundary at Risk**

## II. VaR vs coherent measures of risk

- methodology *BaR* (Boundary at Risk) extends *VaR*, it consists for each quantity, say  $X$ , to define a domain  $D$  and a threshold of probability  $p$  such that  $P\{X \notin D\} \leq p$ .

This way of thinking possesses the natural coherence: if several situations are considered depending on a parameter  $\alpha$  and if

$$P_\alpha\{X \notin D_1\} \leq p$$

for  $\alpha$  in the domain  $D_2$ , if  $\alpha$  becomes random and if the probability that  $\alpha$  be outside  $D_2$  is less than  $q$ , then for the resulting probability one has evidently:

$$P\{\alpha \notin D_2, X \notin D_1\} \leq p + q.$$

The language *BaR* allows to improve a model by taking in account new aleas provided that the new randomized parameters be managed in the same way.

More generally, if two random quantities are considered  $X$  and  $Y$  and if  $N(y, dx)$  is the conditional law of  $X$  given  $Y = y$ . If we know

i) the measure  $N(y, dx)1_{D_1}(x)$  when  $y$  is in  $D_2$

ii) the law of  $Y$  given the event  $Y \in D_2$  i.e. the probability measure  $\frac{1_{D_2}(y)P_Y(dy)}{P_Y(D_2)}$

then we know the joint law of the pair  $(X, Y)$  given the event  $\{X \in D_1\}$  and  $\{Y \in D_2\}$ .



## II. *VaR vs coherent measures of risk*

- VaR is not subadditive in general
- Coherent measures of risk  
(Artzner, Delbaen, et al. 1999)

any risk criterion satisfying the common sense rules when two portfolios are gathered or when a portfolio is multiplied by a deterministic constant, (subadditivity  $C(X_1+X_2) \leq C(X_1)+C(X_2)$ ; homogeneity  $C(\lambda X_1) = \lambda C(X_1)$ ; monotony  $C(X_1) \leq C(X_2)$  if  $X_1 \leq X_2$ ; invariance by translation) is necessarily of the form

$$(1) \quad m(L) = \sup\{\mathbb{E}_P[L] \mid P \in \mathbf{P}\}$$

where  $\mathbf{P}$  is a set of probability laws,  $\mathbb{E}_P$  the expectation symbol under the probability  $P$  and  $L$  the loss.

## II. *VaR vs coherent measures of risk*

**VaR or BaR**

**no tail is used**

**Coherent measures of risk**

**knowledge of the whole  
distribution is required**

**logic of scenarii**

### **III. *The financial crisis : a short analysis***

Trying to keep his bacon happy, Fed Farmer was likely to create more problems...

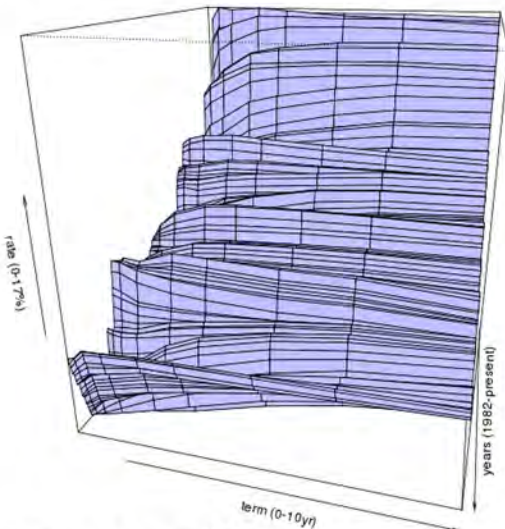
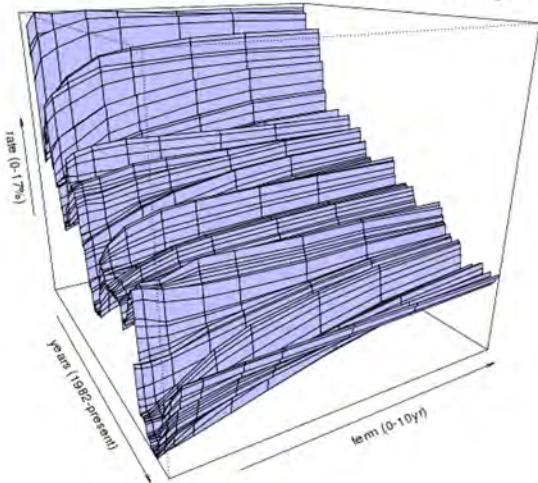


### **III. *Financial crisis : a short analysis***

**The crisis arises when the finance is highly mathematized**

- the «Black-Scholes» revolution and the hedging of options
  - development of derived markets
  - new professional profiles in finance
- term structure and rate models
  - Cox-Ingersoll-Ross, Heath-Jarrow-Morton
  - high dimensional SDE's
  - long term behaviour of agents and calibration today
- credit derivatives
  - markets of mortgage loans
  - credit default swaps (CDS)

# Historical Treasury Yield Curves (1982-present)



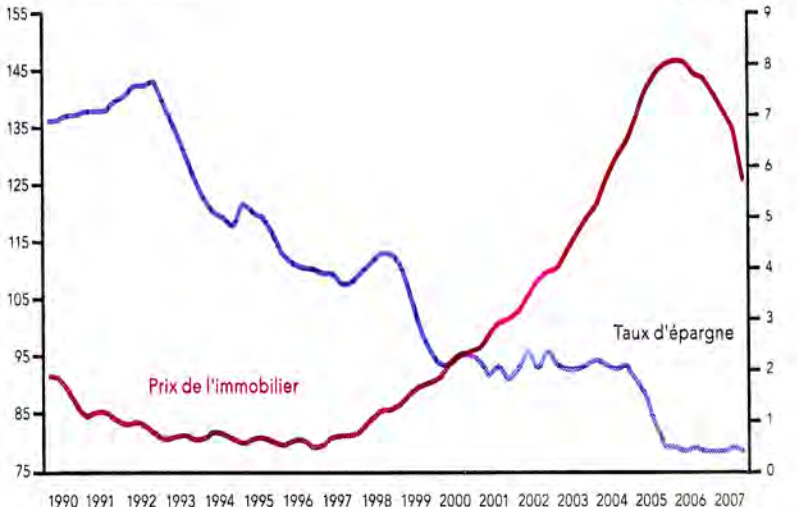
# Interest rates surface



# Prix de l'immobilier et épargne des ménages aux Etats-Unis

Prix de l'immobilier

(indice 100 : moyenne historique)



Taux d'épargne  
des ménages

(en glissement annuel)

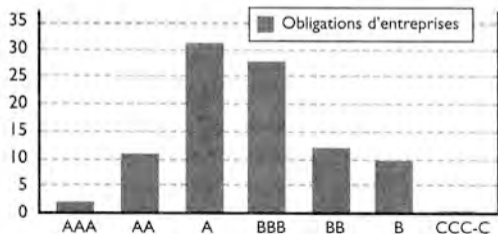
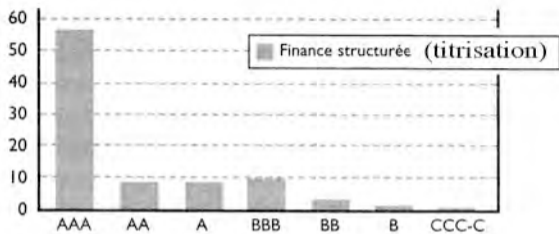
Prix de l'immobilier

Taux d'épargne

1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007

Bureau of Economic Analysis, Bureau of Labor Statistics, Standard and Poor's, MacroMarkets, Haver Analytics, Calculs de l'auteur

# Histogrammes des notations par les agences



Sources: Fitch Ratings, *Financial Times*, 16 août 2007.



# **III. *Financial crisis : a short analysis***

## **Mathematics in the hot seat**

- **Maths have taken away from the «real» economy**
  - **Lionel Jospin (Le Débat jan 2009)**
  - **fundamentals and subjective probabilities**
- **position of some quants and some mathematicians:**
  - **the non-arbitrage principle suggests to take as less as possible risk**
  - **traders are gamblers... possibly crazy**
  - **the ones who do not follow our models aren't rational**
- **on both sides there is a scapegoat (bouc émissaire) maths or crazy traders**

### **III. *Financial crisis : a short analysis***

**It is the meaning of the event that makes the risk**

- Market of risks : an a priori good idea but...**
  - calculations of risks are necessary to price the exchanges**
  - that encapsulates the risks**
- the probabilistic representation of risks as a pair (probability law, function of costs) is fundamentally an deletion of the meaning**
- the meaning of the event changes with what is understood**
  - exemple of discovering new causes**



# ***IV. Chance and meaning***

***two non-miscible fluids***

**Siméon Denis Poisson (1781-1842)**

**Recherches sur la probabilité des jugements**



**Antoine Augustin Cournot (1801-1877)**

**Exposition de la théorie des chances  
et des probabilités (1834)**

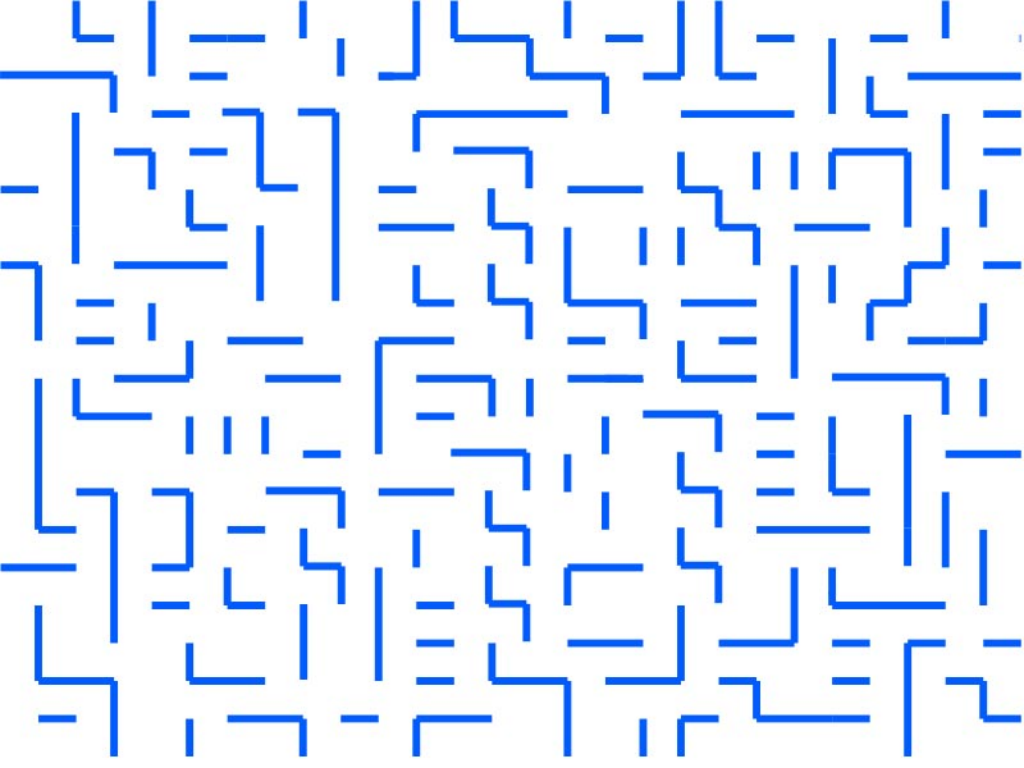
The background of the image is a close-up of water with several oil droplets. The droplets are primarily yellow and orange, with some red ones. The water surface is dark grey, and the droplets are bright and reflective. The overall effect is a colorful, abstract pattern.

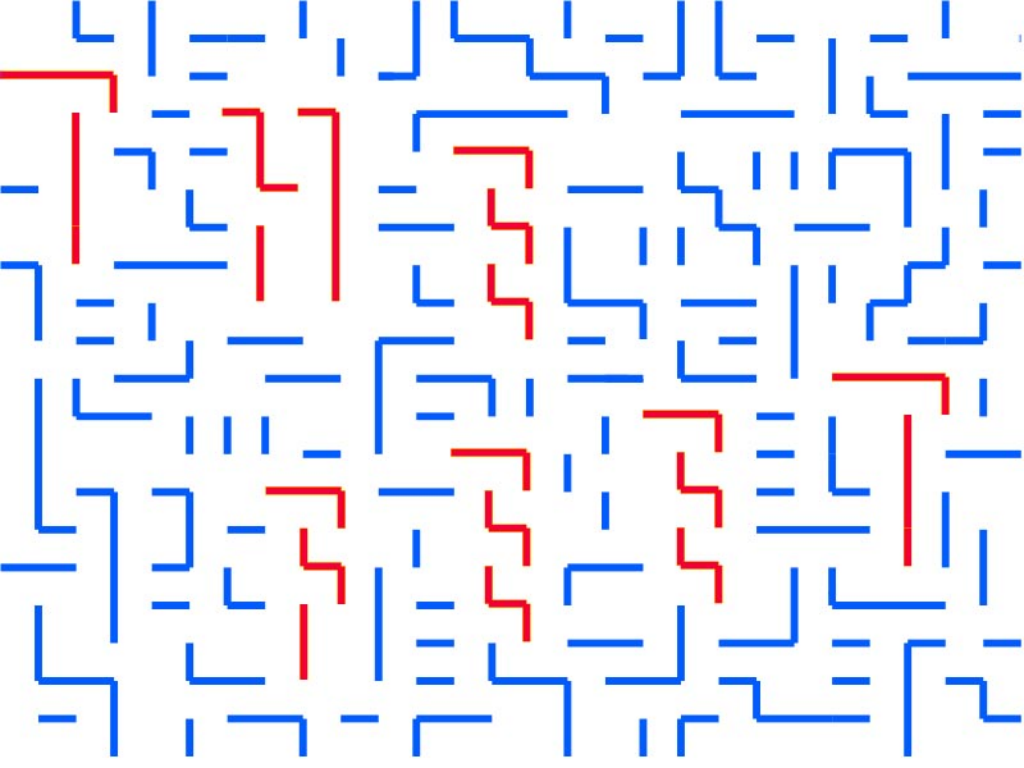
**TRANSITION**

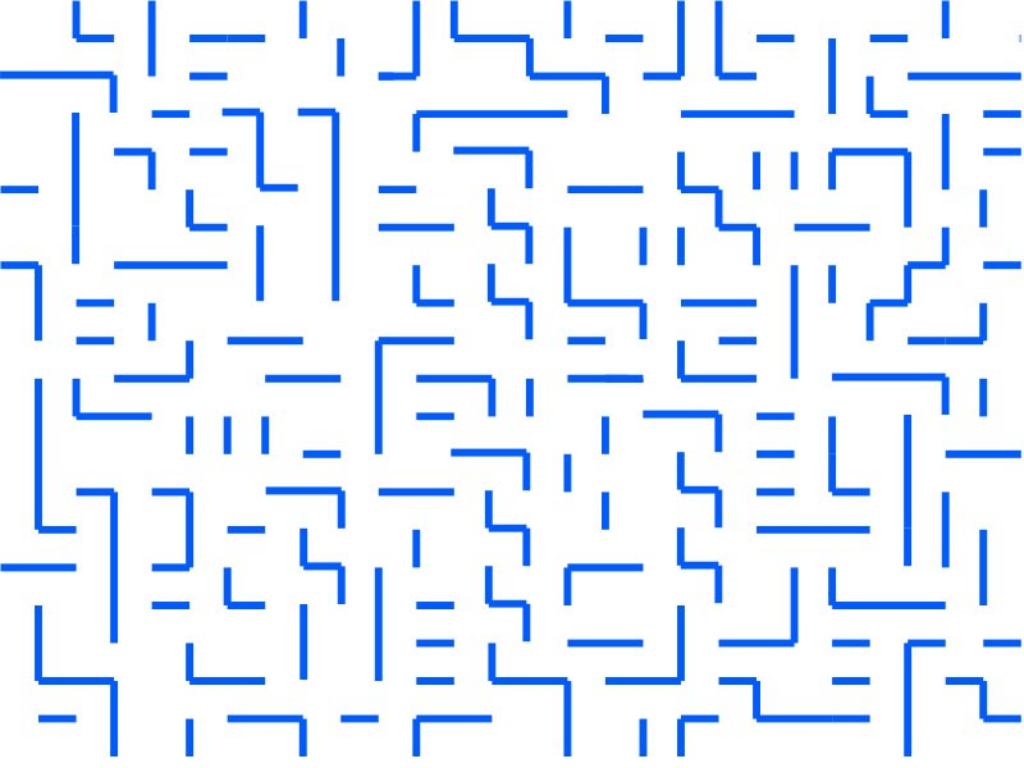
***Chance***



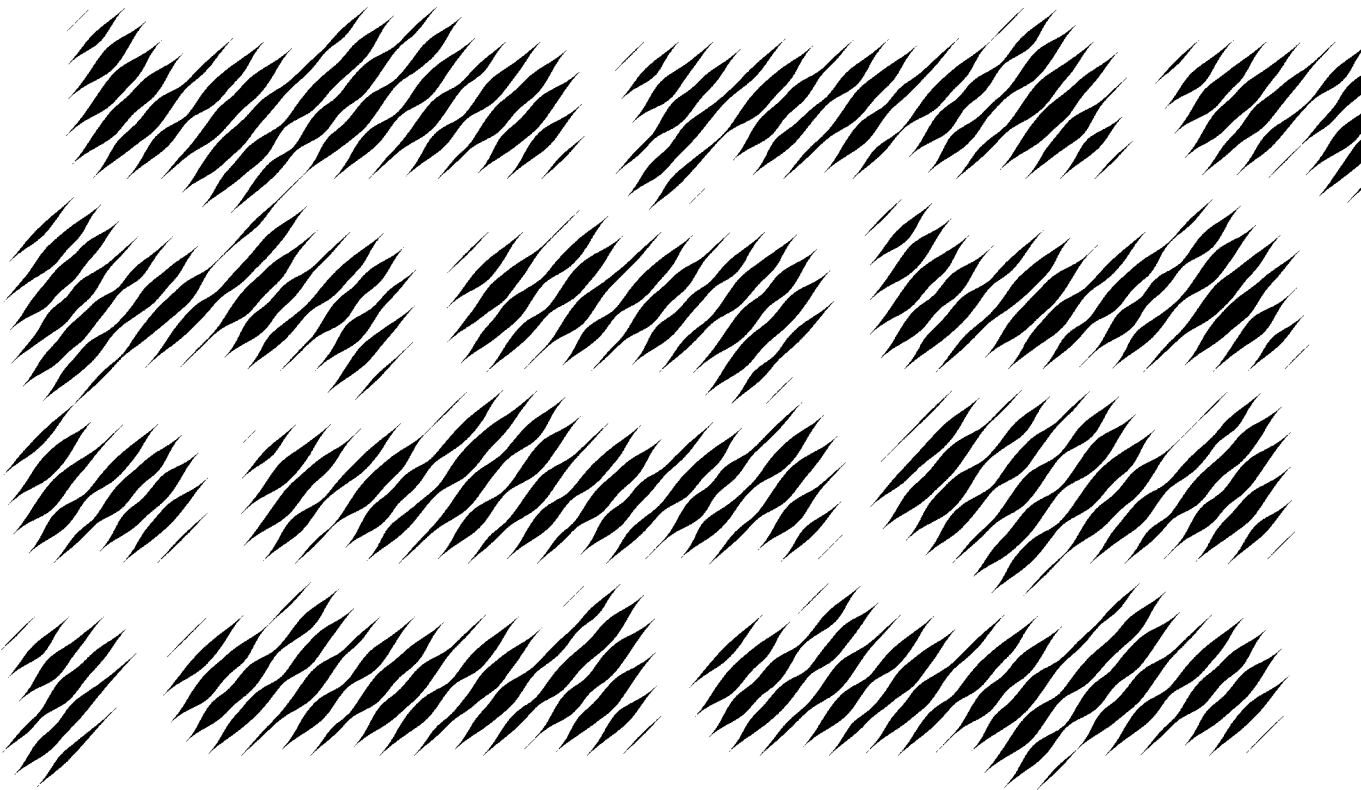
***meaning***

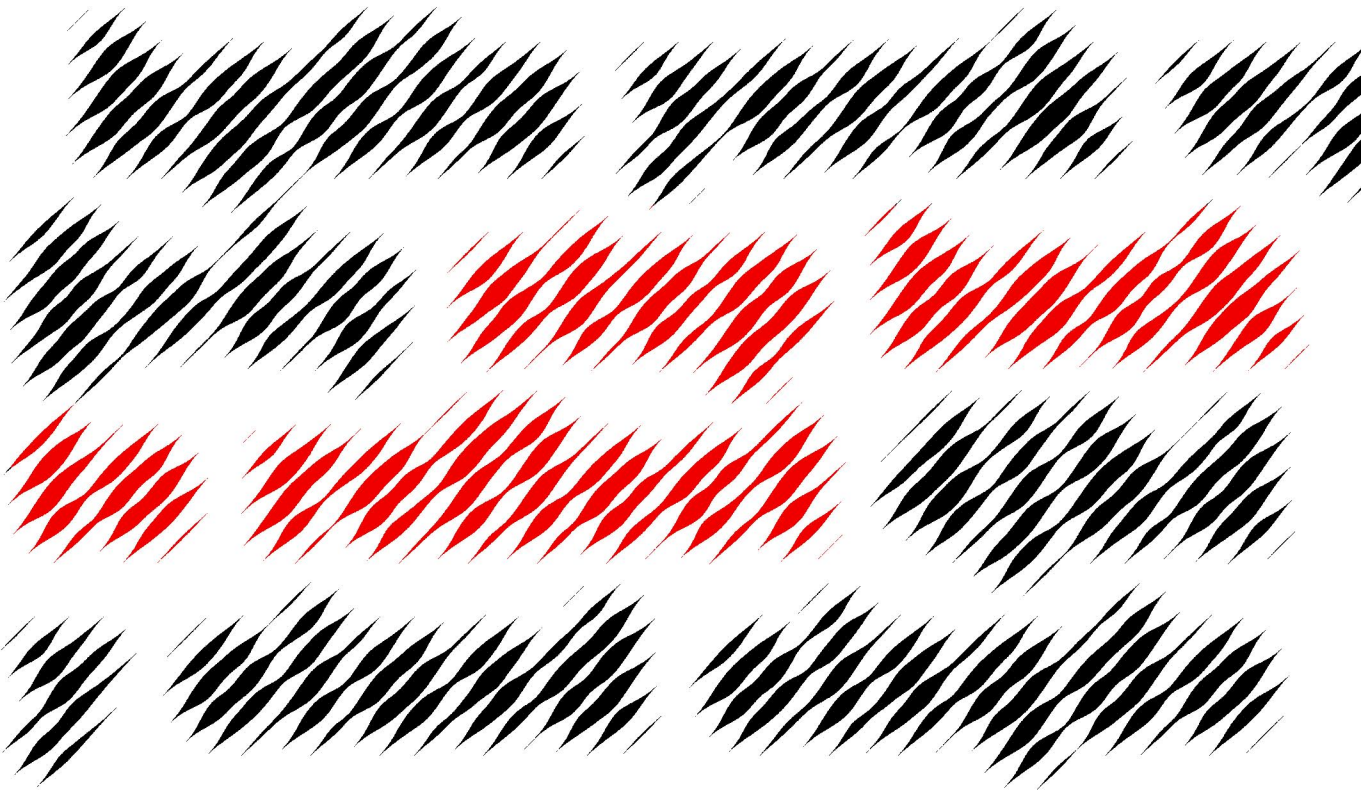


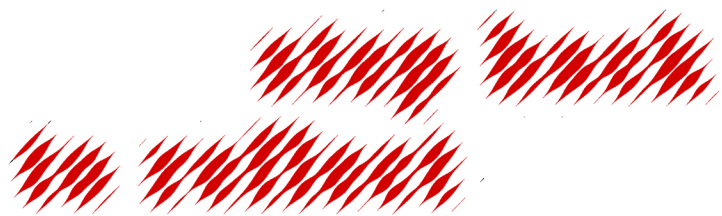


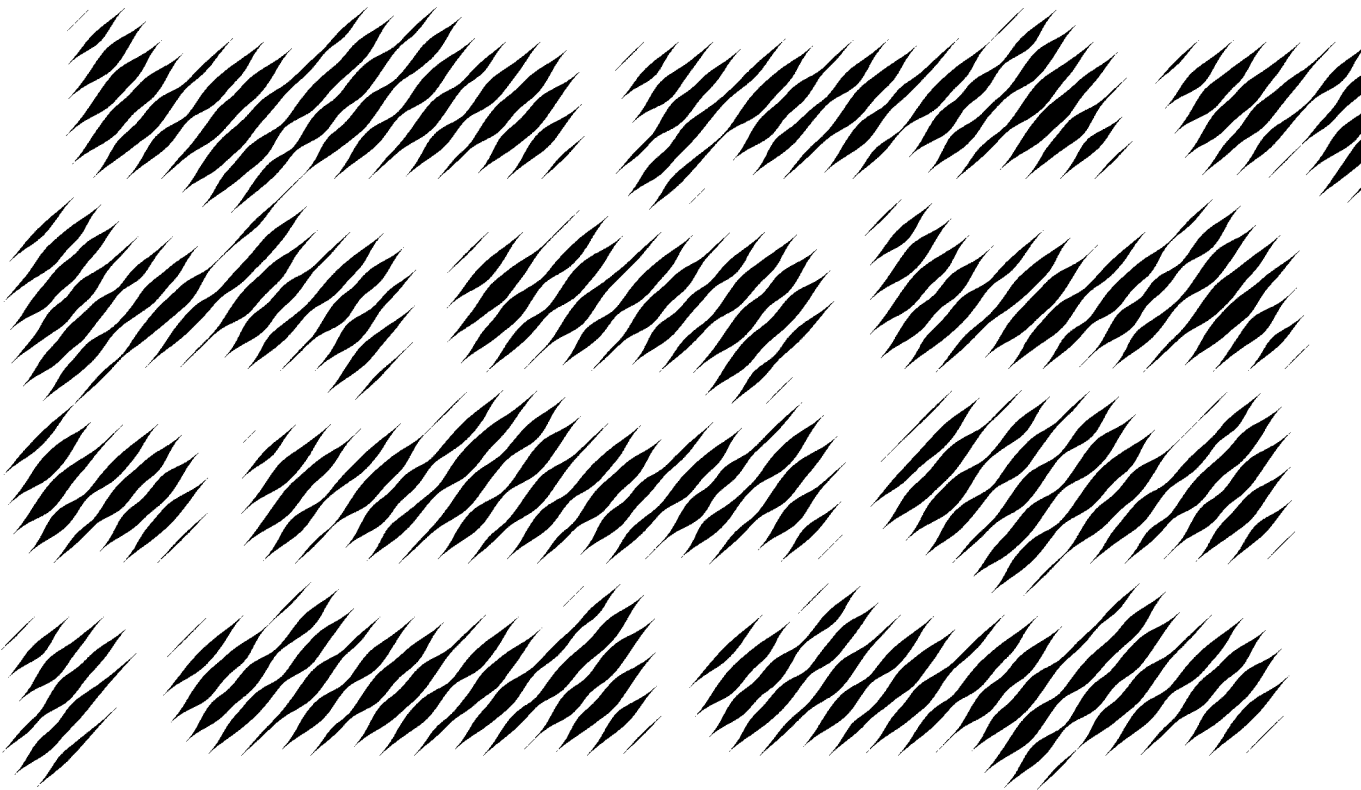


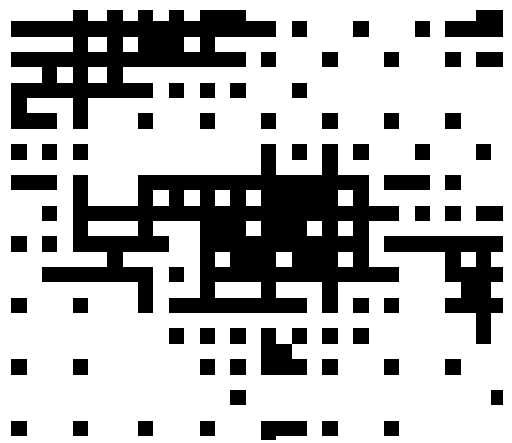




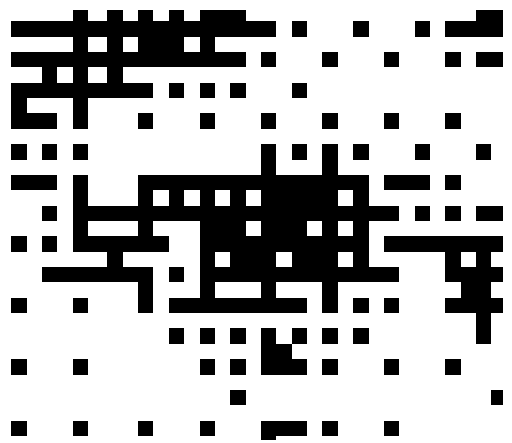






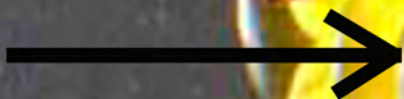






**TRANSITION**

**meaning**



**chance**





**Jugend Styl  
Vienne 1898**



**De Stijl  
Rotterdam 1924**



Frank Lloyd Wright et Bruno Zevi  
A Venise en 1951

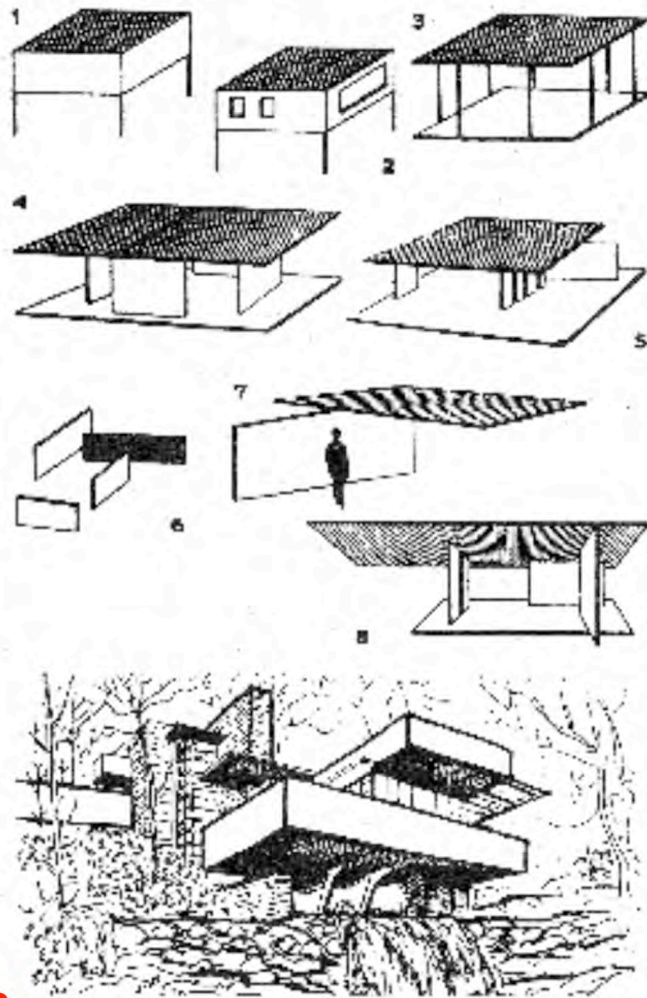


Bruno Zevi théoricien de l'architecture notamment connu des élèves architectes pour son analyse de l'espace intérieur de Saint Pierre de Rome écrit dans *Langage moderne de l'architecture* (1973)

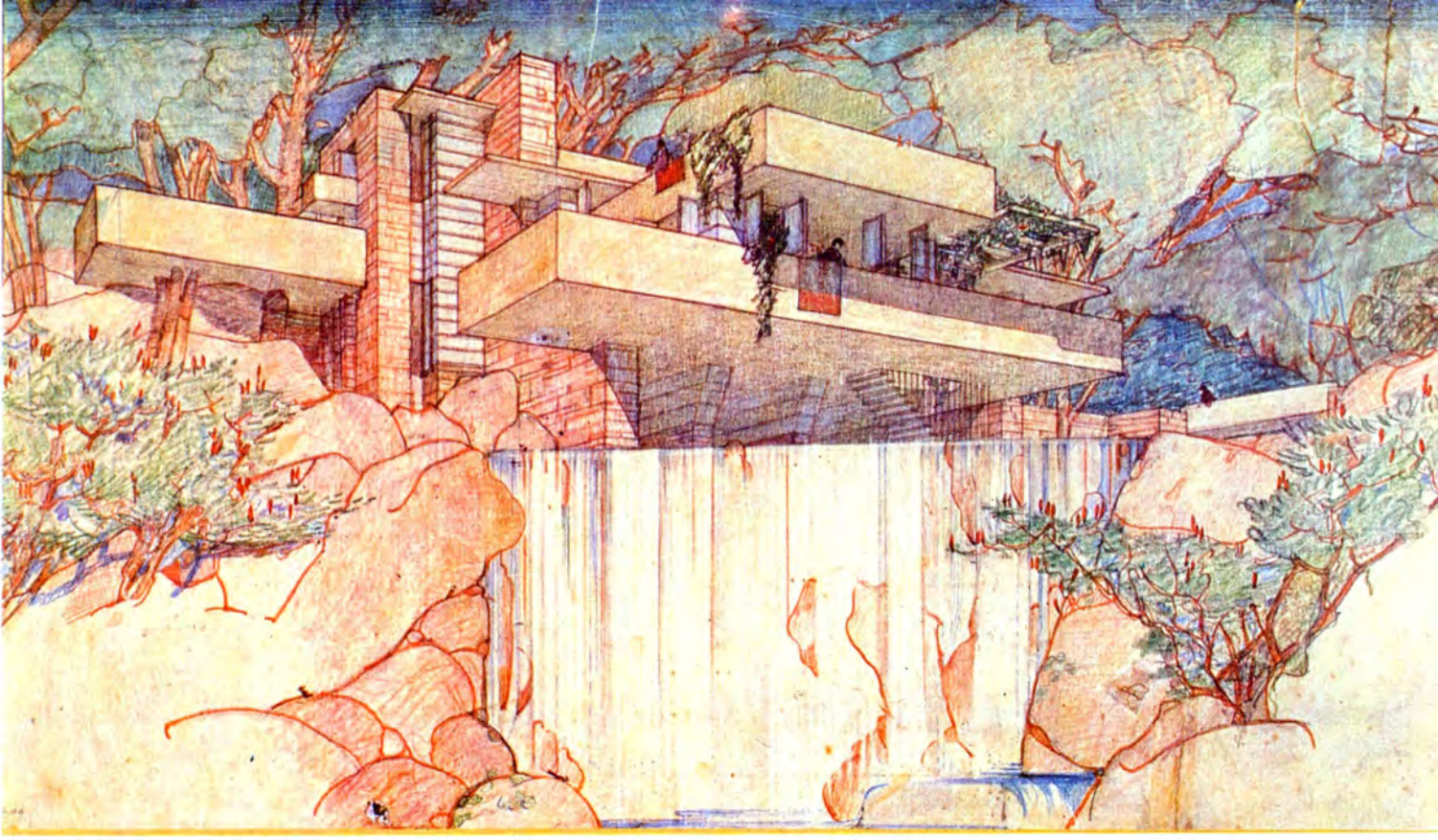
**La symétrie est un invariant du classicisme. Donc la dissymétrie est un invariant du modernisme**

A la question de savoir où situer une fenêtre, une porte, un objet hors des symétries, il répond

**N'importe où ailleurs**



13. Plans qui illustrent une réflexion de Frank Lloyd Wright sur l'intégration de chaque élément d'architecture dans un schéma structural. En haut : croquis de la Maison Kaufmann au sein d'un site sur le conseil de 1916-19; Frank Lloyd Wright, 1916-19, qui réinterprète les sept invariants du langage moderne.

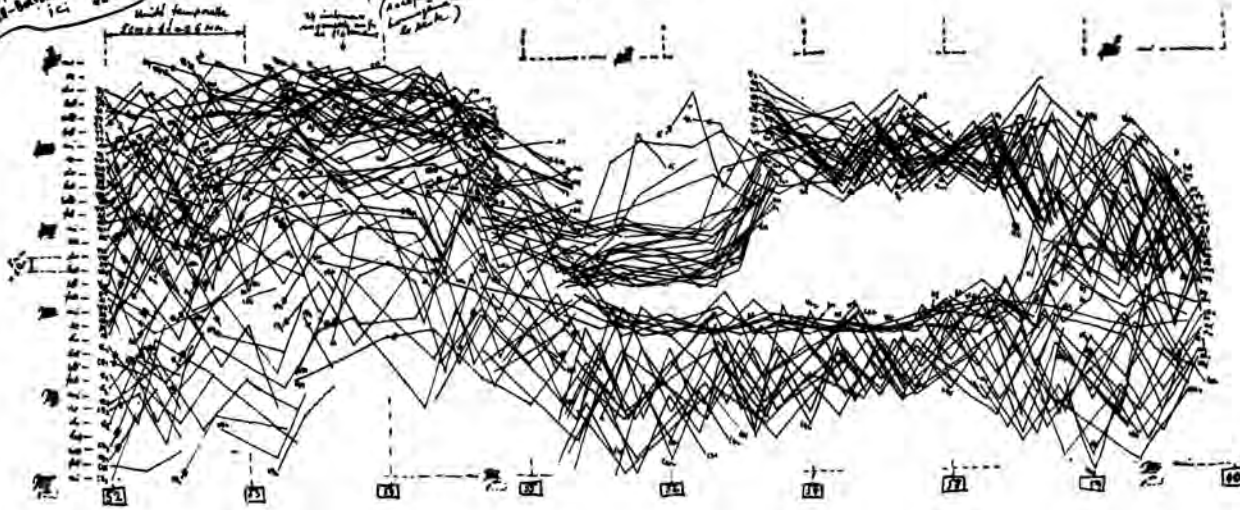


riefen die Leute  
 die gezeichneten  
 Linien für die  
 die Haupt-Beziehungen  
 hier

11/11  
 Piss-gewand

Landwirtsch.  
 Maschinen  
 (A. & S. Fischer  
 Leipzig)

Fibergewebe - Tüllgewebe



# ***V. Back on the tails of distributions***

## ***V. Back on the tails of distributions***

- They change with the interpretation
  - there is no objective unknown tail
  - as soon as a new lecture of the economic complexity appears, the tails vary
- in the case of climate, the meaning of the uncertainties in IPCC models has changed
  - at the beginning : physical unknowns
  - now : psychological and political unknowns
- in a world of pluralism, where the agents attempt to understand what happens,
  - no mathematical modeling of social events may be objective nor universal